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ERRATA

VOLUME 29, PAGE 76. Omit the Corollary.

ON CUBIC FIELDS*.

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I have in this short sketch set myself the task of investigating two problems. The first of these concerns the "außerwesentlichen" primefactors for any given cubic and also higher irrationality. They are, as is known†, characterized by the fact that with a given equation $F(x)=0$

$$F(x) \equiv \prod(x) + p \cdot M(x) \tag{1}$$

where $\prod(x)$ is the product of all irreducible factors of $F(x)$ (mod. p) and for some rational b

$$\prod(b) \equiv 0, \qquad \prod'(b) \equiv 0, \qquad M(b) \equiv 0 \text{ (mod. } p).$$

The general method for determining the ideal primefactors fails for such primes. We shall see how, by modification, it can be used in this case also. The other task is to find any systematic way to decide when two ideals are equivalent, and to construct units. This is done by the construction of chains. It will be seen that each equivalence from the chain gives rise to an equivalence of ideals. The converse theorem is not necessarily true. It is indeed connected with some difficulty to give a method of forming periodic developments with higher irrationalities than quadratic, and therefore it will be necessary to leave the domain $K(1)$, where only the quadratic irrationnalities are periodic. It is however possible to form a continued set of inequalities of approximation in three dimensions and construct chains with coefficients from the cubic field $K(\omega)$ itself, which really are periodic or have, as we shall say, periodic convergence.

As to the former problem we start from the fundamental equation

$$F(x) = x^3 + A_1 x^2 + A_2 x + A_3 = 0, \tag{2}$$

and have to treat the cases when $F(x) \equiv 0$ (mod. p) has a double or a triple root. In the first case

$$F(x) \equiv (x-a)(x-b)^2 + p^i \cdot M(x),$$

* Received March 19, 1928.

† Bachmann, P.: Allgemeine Arithmetik der Zahlenkörper, p. 277.